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LONG-TERM STRENGTH OF METALS AND CREEP EQUATIONS BASED ON THE COULOMB–MOHR CRITERION

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It is proposed to construct long-term strength and creep relations for metals on the basis of the Coulomb–Mohr criterion. The creep equations and the long-term strength criterion for plane stress are analyzed in detail. Results of long-term strength calculations are compared with data of experiments with metallic materials. It is established that theoretical and experimental results are in satisfactory agreement.

Key words: *plasticity, creep, long-term strength of metals, Coulomb–Mohr yield criterion.*

Over the past 50 years, extensive studies of the creep and long-term strength of metals in a complex stress state have been performed. An overview and analysis of some experimental studies can be found in [1–11].

To determine the rupture time of a structural member in a complex stress state under creep conditions, it is necessary to choose the long-term strength criterion corresponding to the test conditions. The chosen criterion is used to determine the equivalent stress states leading to rupture for the same time and to calculate this time using data of a simple test (in uniaxial tension, compression or pure shear). As a rule, the creep equations are based on a version of plasticity theory, and the long-term strength criterion on strength theory. This is due to the fact that by the time of publication of the first papers on the technical theories of creep, the classical plasticity equation and the basic strength theories had already been formulated [1].

In plasticity theory, the Coulomb–Mohr criterion is used for soils and rocks. This is likely the reason why this criterion has not currently been used to study the long-term strength of metals.

The applicability of the Coulomb–Mohr criterion to the plastic deformation of metals has been proven by various experimental results [12]. It has been established [13] that, for metals, rocks, soils, and loose media, this criterion provides adequate accuracy in determining the limiting stresses and the rupture directions identified with the characteristics of the equations for the rate field. For an arbitrary stress state, the Coulomb–Mohr criterion is written as

$$\max_n \Big\{ |\tau_n| + \sigma_n \tan\varphi \Big\} = C, \tag{1}$$

where τ_n and σ_n are the tangential and normal stresses in a plane with normal $\boldsymbol{n}$, φ is the internal friction angle, and C is the cohesion.

Numbering the principal axis by I, II, and III so as to satisfy the inequalities

$$\sigma_{\rm I} > \sigma_{\rm II} > \sigma_{\rm III}, \tag{2}$$

we write criterion (1) as

$$\frac{\sigma_{\rm I} - \sigma_{\rm III}}{2\cos\varphi} + \frac{\sigma_{\rm I} + \sigma_{\rm III}}{2} \tan\varphi = C. \tag{3}$$

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For various dependences of the cohesion C on the rupture time t_* in (1) or (3), we obtain different versions of the Coulomb–Mohr long-term strength criterion. If the left side of equality (3) is used as an equivalent stress σ_e, then, for the power-law dependence with zero and nonzero creep strength σ_0, the rupture time is defined, respectively, by the formulas

$$t_* = A^n \sigma_e^{-n}, \qquad t_* = A^n (\sigma_e - \sigma_0)^{-n},$$

where A and n are the material characteristics calculated from results of experiments. From these formulas, for the long-term strength criterion (3) we have, respectively,

$$C(t_*) = A/t_*^{1/n}, \qquad C(t_*) = \sigma_0 + A/t_*^{1/n}.$$

From results of two experiments with uniaxial tension and torsion, using criterion (3) it is possible to obtain the internal friction angle φ and the long-term strength $C = C(t)$:

$$\sin\varphi = 2\tau_s/\sigma_t - 1, \qquad C = (1 + \sin\varphi)\sigma_t/(2\cos\varphi) \tag{4}$$

(τ_s and σ_t are the long-term strength for torsion and uniaxial tension, respectively).

Experimental results of Johnson [5] show that for long-term behavior of a material at high temperatures, the maximum normal stresses $\sigma_{e1} = \sigma_1$ can serve as the strength criterion. Kats [14] processed experimental data using the normal stress intensity as the criterion:

$$\sigma_{e2} = \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2}/\sqrt{2}. \tag{5}$$

Sdobyrev [7, 8] proposed a criterion which is in good agreement with experimental data and is represented as the half-sum of the stress intensity and the maximum normal stresses:

$$\sigma_{e3} = (\sigma_{e2} + \sigma_{e1})/2.$$

The possibility of using this criterion is confirmed by the results of experiments of Trunin [10]. For the rupture of metals under long-term constant loading, Lebedev [11] proved the applicability of a generalized criterion which includes the quantities σ_{e2} and σ_{e1} and a certain coefficient χ dependent on the properties of the material:

$$\sigma_{e4} = \chi(\sigma_{e2} - \sigma_{e1}) + \sigma_{e1}. \tag{6}$$

The coefficient χ can be defined as a quantity that characterizes the contribution to the macrorupture from shear deformation, which creates favorable conditions for loosening of the material in. For $\chi = 0$, when the rupture is determined only by the strength of grain boundaries, criterion (6) becomes the Johnson criterion [5]. The coefficient $\chi = 1$ if rupture results from shear processes inside a grain; in this case, criterion (6) coincides with the Kats criterion (5). The coefficient $\chi = 0.5$ if the softening effect of shear deformation is equivalent to the effect of normal stresses; in this case, criterion (6) coincides with the Sdobyrev criterion [7, 8].

For creep, as well as for plasticity [12, 13], using the Coulomb–Mohr criterion (3), in view of (4), is justified. For $\varphi = 0$, the intracrystalline rupture mechanism is dominant, and, in this case, criterion (3) coincides with the maximum tangential stress criterion. For $\varphi = \pi/2$, the intercrystalline rupture mechanism is dominant, and, in this case, criterion (3) coincides with the Johnson criterion [5].

Below, we give the results of experimental studies [7, 8, 10] of the long-term strength of thin-walled cylindrical samples loaded by a tensile force and a twisting moment.

The tests in [10] were performed for 1Kh18N12T austeinitic steel at a temperature of 610°C and for 15Kh1M1F pearlitic steel at a temperature of 570°C. In both cases, three series of experiments with tubular samples were conducted: 1) uniaxial tension ($\sigma_x = \sigma$ and $\tau_{xy} = 0$); 2) pure torsion ($\sigma_x = 0$ and $\tau_{xy} = \tau$); 3) joint tension and torsion ($\sigma_x = \sigma$ and $\tau_{xy} = \sigma/2$). As a rule, experimental data are processed as follows: a combination of stress tensor invariants is used as an equivalent stress σ_e, a power-law or exponential dependence of t_* on σ_e ($t_* = A\sigma_e^{-m}$ or $t_* = B \cdot 10^{-\sigma_e/n}$) is chosen, and, accordingly, long-term strength diagrams are constructed in logarithmic coordinates ($\log t_*$, $\log \sigma_e$) or semilogarithmic coordinates ($\log t_*$, σ_e) and are approximated by straight lines. The straight line equation for each value of the stress σ_e is determined by the least-squares method using the dispersion D of the distances from the experimental points to this straight line as a characteristic of the scatter of experimental data. The equivalent stress σ_e corresponding to the minimum dispersion $D_{\min}$ is chosen as the long-term strength criterion. In [6], this technique was employed to perform statistical processing of all known experimental data and long-term strength criteria were obtained for various materials under various test conditions.

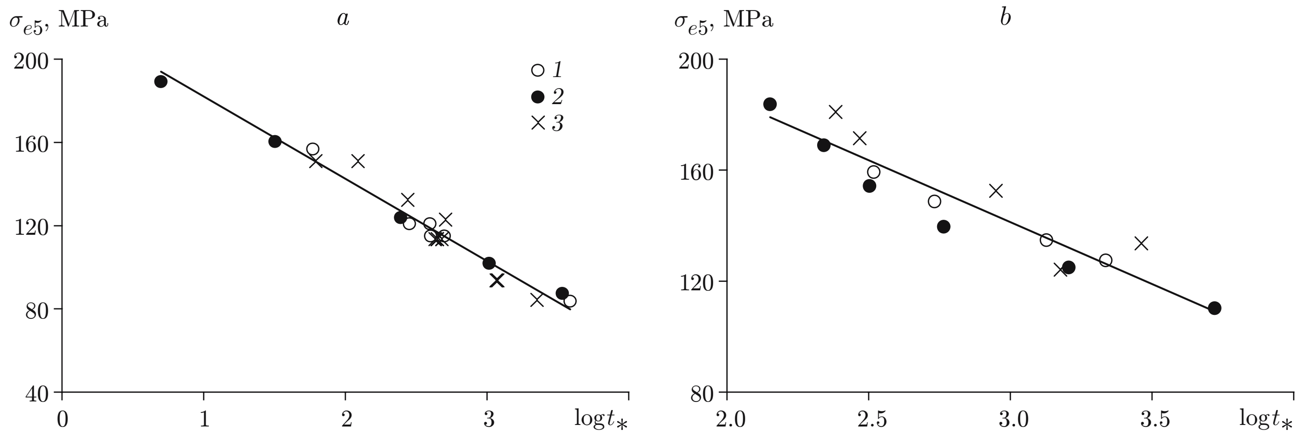

Fig. 1. Results of processing of the experimental data of [10] for 1Kh18N12T steel samples (a) and 15Kh1M1F steel samples (b) using the Coulomb–Mohr criterion: 1) torsion; 2) tension; 3) tension and torsion; the curve is a least-squares approximation of experimental data.

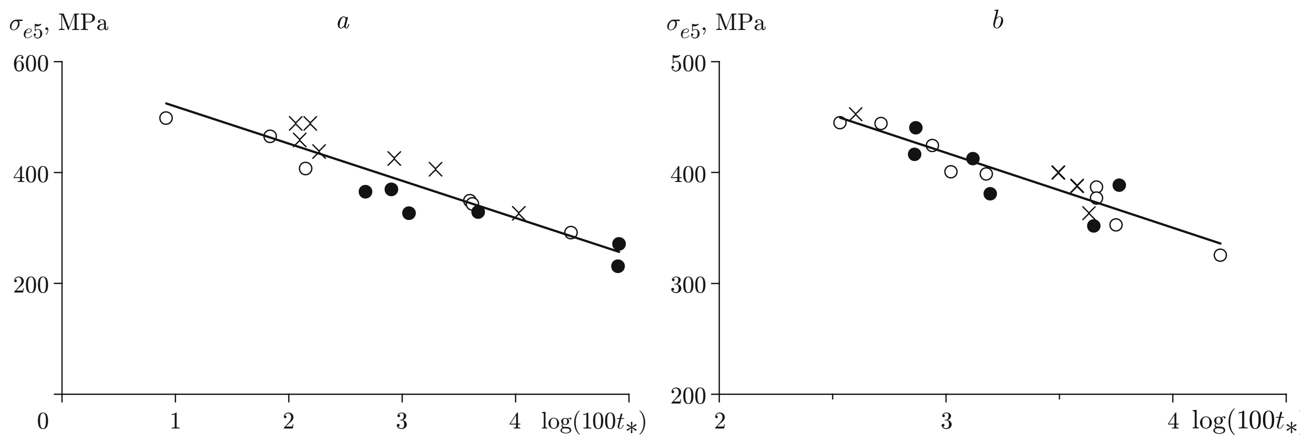

Fig. 2. Results of processing of the experimental data of [7] (a) and [8] (b) using the Coulomb–Mohr criterion (notation the same as in Fig. 1).

The calculations for austenitic and pearlitic steels were performed using the same technique. Experimental points ($\log t_*$, τ_s) for torsion and ($\log t_*$, σ_t) for uniaxial tension were plotted in semilogarithmic coordinates; straight lines were drawn through the points using the least-squares method. Then, using formula (4) by averaging for 100 and 1000 hr, the internal friction angle $\varphi = 23.2°$ was found for austenitic steel and $\varphi = 22.6°$ for pearlitic steel. Figure 1 gives the results of processing of the experimental data of [10] in semilogarithmic coordinates ($\log t_*$, σ_{e5}), where σ_{e5} is determined by the Coulomb–Mohr criterion (3). The logarithm of the rupture time t_* (time is in hours) is plotted on the abscissa.

The experimental data of [7, 8] for ÉI437B alloy were processed similarly; the results are presented in semilogarithmic coordinates ($\log (100t_*)$, σ_{e5}) in Fig. 2. For ÉI 437B [7] alloy, the internal friction angle is $\varphi = 34°$, and for ÉI437B alloy of another heat (51364 heat) [8], $\varphi = 28°$. For each criterion of σ_{ej} ($j = 1, \ldots, 5$), the standard deviation of the experimental points from the linear dependence constructed by the least-squares method was determined:

$$\Delta_j = \sqrt{D_j}, \qquad D_j = \frac{1}{n-1} \sum_{i=1}^{n} (\sigma_{ej}^i - \sigma_{ej}(t_i))^2, \qquad j = 1, \ldots, 5.$$

Here D_j ($j = 1, \ldots, 5$) is the dispersion of the distances from the experimental points to the corresponding linear long-term strength dependences.

TABLE 1

Relative Standard Deviation of Experimental Data
from the Linear Dependence According to Various Criteria

Material	Δ_1/Δ [5]	Δ_2/Δ [14]	Δ_3/Δ [7]	Δ_4/Δ [11]	χ	Δ_5/Δ	φ, deg
1Kh18N12T steel [10]	3.86	3.25	1.31	1.28	0.58	1.00	23.2
15Kh1M1F steel [10]	3.38	2.35	1.34	1.21	0.61	1.00	22.6
ÉI437B alloy [7]	1.39	2.31	1.14	1.00	0.275	1.03	34,0
ÉI437B alloy [8]	4.15	4.14	1.11	1.11	0.50	1.00	28.0

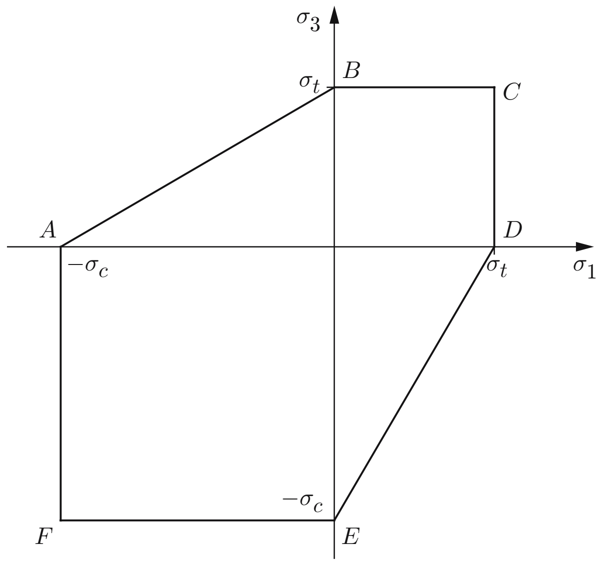

Fig. 3. Coulomb–Mohr criterion in the stress plane (σ_1, σ_3).

The results of processing of the experimental data of [7, 8, 10] using different long-term strength criteria are presented in Table 1. For each criterion, the table gives the standard deviation Δ_j/Δ, where $\Delta = \min\limits_j \Delta_j$, $j = 1, \ldots, 5$. In contrast to σ_{e1}, σ_{e2}, and σ_{e3}, the criteria σ_{e4} and σ_{e5} depend on the material constants χ and φ, respectively. The values of χ and φ given in Table 1 were calculated using the same technique for each series of experiments: the average values of χ and φ for each material were first determined from two values of the rupture time, and then, if necessary, they were refined by using the minimum condition for the dispersion D.

A comparison of the results given in Table 1 and Figs. 1 and 2 leads to the conclusion that the Coulomb–Mohr long-term strength criterion provides better agreement with the experimental data of [7, 8, 10] than the most widely used criteria listed above.

Let us consider the creep equations based on the dilatation-shear theory of plasticity [12]. If inequality (2) is satisfied, the creep strain rate tensor components in the principal stress axes obey the following relations:

$$\dot{e}_{\mathrm{I}} = \frac{\alpha(1+\sin\varphi)+\cos\varphi}{2}\,\dot{\gamma}_2, \qquad \dot{e}_{\mathrm{II}} = 0, \qquad \dot{e}_{\mathrm{III}} = \frac{\alpha(1-\sin\varphi)+\cos\varphi}{2}\,\dot{\gamma}_2 \tag{7}$$

(α is the dilation coefficient). For short-term loading of an ideally plastic material, $\dot{\gamma}_2$ is an unknown parameter, which is found by solving a particular problem; in the case of a strain-hardening material, $\dot{\gamma}_2$ is determined by the stress level attained and stress increments [12]. For long-term loading under creep conditions, from yield and hardening theories we have, respectively,

$$\dot{\gamma}_2 = f_1(\tau_2, t), \qquad \dot{\gamma}_2 = f_2(\gamma_2, \tau_2).$$

Let the principal stress $\sigma_y = \sigma_2 = 0$, and let σ_1 and σ_3 be the other two principal normal stresses. Then, the intersection of the Coulomb–Mohr pyramid with the stress plane (σ_1, σ_3) forms an irregular hexagon $ABCDEF$ (Fig. 3), whose sides and vertices belong to the faces and edges of a pyramid, respectively. Depending on the sign and value of the principal stresses σ_1 and σ_3, the limiting state can be attained in various areas; therefore, the plane stress equations for the Coulomb–Mohr criterion are different for different loading modes. If the stresses σ_1 and σ_3 have opposite signs, the limiting state is attained in areas perpendicular to the planes (x, z) and passing through the axes corresponding to the second principal direction. In this case, in the plane (x, z) there are two families of characteristics for which the following relations are valid:

$$\begin{aligned} \frac{dz}{dx} &= \tan\,(\theta - \psi), \qquad \cot\,\varphi \ln\Big(1 - \frac{\sigma}{C}\tan\,\varphi\Big) + 2\theta = \text{const} = \xi, \\ \frac{dz}{dx} &= \tan\,(\theta + \psi), \qquad \cot\,\varphi \ln\Big(1 - \frac{\sigma}{C}\tan\,\varphi\Big) - 2\theta = \text{const} = \eta. \end{aligned} \tag{8}$$

Here $\psi = \psi_\sigma = \pi/4 + \varphi/2$ is the angle between the characteristic of the first set (α-line) and the first principal direction of the stress tensor σ_1; θ is the angle between the direction of σ_1 and the x axis: $\tan\,2\theta = 2\tau_{xz}/(\sigma_x - \sigma_z)$. In the following, equations of characteristics of the form (8) are also used for rates. Relations (8) are valid for mode DE, where $\sigma_1 > \sigma_2 > \sigma_3$, and for mode AB, where $\sigma_3 > \sigma_2 > \sigma_1$.

The equations for the rate field in the given loading modes for the nonassociated model [12] were obtained for the first time in [13]:

$$\begin{aligned} \tan\,(2\theta)\,\frac{\partial v_x}{\partial x} - \frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} - \tan\,(2\theta)\,\frac{\partial v_z}{\partial z} &= 0, \\ (a\cos{(2\theta)} - b)\,\frac{\partial v_x}{\partial x} + (a\cos{(2\theta)} + b)\,\frac{\partial v_z}{\partial z} &= 0 \end{aligned} \tag{9}$$

($a = 1 + \alpha\tan\,\varphi$ and $b = \alpha/\cos\varphi$). The given system of differential equations is a hyperbolic system, and the directions of its characteristics are uniquely determined by the internal friction angle φ and the dilatation coefficient α. For $\alpha = \tan\,\varphi$, the characteristics of Eqs. (9) for the rate field coincide with the characteristics of the stress field equations (8).

Let us consider mode CD, for which $\sigma_1 = \sigma_t > \sigma_3 > \sigma_2 = 0$ (σ_t is the long-term strength for uniaxial tension). In this case, in deriving equations for stresses and rates on the basis of (2) and (3), one needs to set $\sigma_{\text{I}} = \sigma_1$, $\sigma_{\text{II}} = \sigma_3$, and $\sigma_{\text{III}} = \sigma_2$. We use the following notation:

$$(\sigma_1 + \sigma_3)/2 = \sigma, \qquad (\sigma_1 - \sigma_3)/2 = \sigma_t - \sigma. \tag{10}$$

Using the formulas of transformation of the stress components and notation (10), we obtain

$$\sigma_x = \sigma(1 - \cos 2\theta) + \sigma_t \cos 2\theta, \quad \tau_{xz} = (\sigma_t - \sigma)\sin 2\theta, \quad \sigma_z = \sigma(1 + \cos 2\theta) - \sigma_t \cos 2\theta.$$

Substituting the stress components into the equilibrium equations and performing some transformations, we have [15]

$$\begin{aligned} \sin{(\theta)}\,\frac{\partial\theta}{\partial x} - \cos{(\theta)}\,\frac{\partial\theta}{\partial z} &= 0, \\ \sin{(2\theta)}\,\frac{\partial \ln{(\sigma - \sigma_t)}}{\partial x} - (1 + \cos 2\theta)\,\frac{\partial \ln{(\sigma - \sigma_t)}}{\partial z} + 2\,\frac{\partial\theta}{\partial x} &= 0. \end{aligned} \tag{11}$$

For the first equation of system (11), the system of differential equations for the vector lines is written as

$$\frac{dx}{\sin\theta} = \frac{dz}{-\cos\theta} = \frac{d\theta}{0}, \tag{12}$$

which is easily integrated: $\theta = \text{const} = C_1$ and $z + x\cot\,\theta = C_2$.

Thus, the general solution of the first equation of system (11) is given by

$$z + x\cot\,\theta = \Phi(\theta), \tag{13}$$

where $\Phi(\theta)$ is an arbitrary function determined from specified boundary conditions.

For the second equation of system (11), the system of differential equations for the vector lines is written as

$$\frac{dx}{\sin 2\theta} = \frac{dz}{-(1 + \cos 2\theta)} = \frac{d\ln{(\sigma - \sigma_t)}}{-2\,\partial\theta/\partial x}. \tag{14}$$

In the plane (x, z), system (14) has the same set of characteristics as system (12), and, hence, system (11) is a parabolic system. Along the characteristic of system (14), we have

$$d \ln (\sigma - \sigma_t) = -2 \frac{\partial \theta}{\partial x} \frac{dx}{\sin 2\theta}.$$

Integration of this relation in view of (13) yields the general solution of the second equation of system (11):

$$\sigma = \sigma_t + \frac{\Psi(\theta)}{2x + (1 - \cos 2\theta)\Phi'(\theta)}.$$

Let us derive equations for the rate field for mode CD. Using (7), we determine

$$\dot{e}_x = \dot{e}_1(1 + \cos 2\theta)/2, \qquad \dot{e}_z = \dot{e}_1(1 - \cos 2\theta)/2, \qquad \dot{\gamma}_{xz} = \dot{e}_1 \sin 2\theta$$

in arbitrary coordinates (x, z). Eliminating $\dot{e}_1$ from these relations, we obtain the following system of equations for the rates:

$$\tan (2\theta) \frac{\partial v_x}{\partial x} - \frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} - \tan (2\theta) \frac{\partial v_z}{\partial z} = 0,$$

$$(1 - \cos 2\theta) \frac{\partial v_x}{\partial x} + (1 + \cos 2\theta) \frac{\partial v_z}{\partial z} = 0.$$

This system of differential equations is a parabolic system and has one characteristic direction, which coincides with the direction of σ_3. The equation of the characteristic and the relation on it are written as

$$\frac{dz}{dx} = -\cot \theta = \tan (\theta + \pi/2), \qquad dv_3 + v_1 \, d\theta = 0,$$

where v_1 and v_3 are the projections of the rate vector onto the principal stress axes.

Let us consider mode AF, for which $\sigma_2 = 0 > \sigma_3 > \sigma_1 = -\sigma_c$. Here $\sigma_c = \sigma_t(1 + \sin\varphi)/(1 - \sin\varphi)$ is the long-term strength for uniaxial compression. Similarly, one can obtain the general solution for the stresses in mode AF:

$$z + x \cot \theta = \Phi(\theta), \qquad \sigma = -\sigma_c + \frac{\Psi(\theta)}{2x + (1 - \cos 2\theta)\Phi'(\theta)},$$

and to show that the characteristics of the equations for the rate field and the relations on them are identical for modes AF and CD.

Let us consider mode BC, for which $\sigma_3 = \sigma_t > \sigma_1 > \sigma_2 = 0$. For this mode, the general solution for the stresses is given by

$$z - x \tan \theta = \Phi(\theta), \qquad \sigma = \sigma_t + \frac{\Psi(\theta)}{2x + (1 + \cos 2\theta)\Phi'(\theta)}. \tag{15}$$

In this case, the system of differential equations for the rate vector components is written as

$$\tan (2\theta) \frac{\partial v_x}{\partial x} - \frac{\partial v_z}{\partial x} - \frac{\partial v_x}{\partial z} - \tan (2\theta) \frac{\partial v_z}{\partial z} = 0,$$

$$(1 + \cos 2\theta) \frac{\partial v_x}{\partial x} + (1 - \cos 2\theta) \frac{\partial v_z}{\partial z} = 0.$$

This system is a parabolic system, whose characteristic coincides with the direction of σ_1. The equation of the characteristic and the relation on it are written as

$$\frac{dz}{dx} = \tan \theta, \qquad dv_1 - v_3 \, d\theta = 0. \tag{16}$$

For loading in mode EF, we have $\sigma_2 = 0 > \sigma_1 > \sigma_3 = -\sigma_c$. For this mode, as well as for mode BC, the characteristics of the stress and rate equations coincide with the direction of σ_1. The relation on the characteristic for stresses is obtained from (15) by replacing σ_t by $-\sigma_c$. The relations for rates in mode EF coincide with (16).

The equations of an inelastic creeping body for the rate field at the edges of the Coulomb–Mohr pyramid (modes B and D in Fig. 3) coincide with Eqs. (9) but the quantities a and b have different values. This type of loading includes uniaxial tension. The angle between the characteristic of the equations for the rate field and the first principal direction of the stress tensor is found from the formula [13]

$$\cos 2\psi = -\frac{b}{a} = -\frac{\cos\varphi + \alpha(3 + \sin\varphi)}{3\cos\varphi + \alpha(3 + \sin\varphi)}. \tag{17}$$

For modes A and E, which include uniaxial compression as a particular case, the angle ψ is determined from the formula

$$\cos 2\psi = \frac{\cos\varphi - \alpha(3 - \sin\varphi)}{3\cos\varphi - \alpha(1 - 3\sin\varphi)}. \tag{18}$$

Results of experiments on rupture of steel tubes loaded by an axial force and internal pressure under creep conditions are presented in [1]. If the maximum tensile stress is tangential, the cracks are longitudinal. If the maximum tensile stress in a tube is axial, the cracks are predominantly annular. Results of uniaxial compression experiments with prismatic specimens of saliferous rock under long-term loading in creep with subsequent rupture are given in [16]. It has been established that in the case of long-term loading, rupture is manifested in increasing strain rate and formation of longitudinal cracks but the sample does not lose the load bearing ability. The experimental data agree with the results of calculations using formulas (17) and (18) in two cases: 1) if one sets $\alpha = \tan\varphi$ and $\varphi = \pi/2$; 2) if one sets $\alpha = \cos\varphi/(1 - \sin\varphi)$. In both cases, from (17) and (18), we obtain $\psi = \pi/2$.

The above comparison of theoretical and experimental results on the long-term loading of various materials under creep conditions allows one, using the Coulomb–Mohr criterion, to determine the limiting stresses and the rupture directions which coincide with the characteristic directions of the equations of an inelastic creeping body for the rate field.

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